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# Exotic composites: the decay of deficit angles in global-local monopoles

**Ana Achúcarro**
*Lorentz Institute of Theoretical Physics, Leiden University*
*2333 RA Leiden, The Netherlands, and*
*Department of Theoretical Physics, University of the Basque Country UPV-EHU*
*48080 Bilbao, Spain*
*E-mail:* achucar@lorentz.leidenuniv.nl

**Betti Hartmann**
*School of Engineering and Science, International University Bremen (IUB)*
*28725 Bremen, Germany*
*E-mail:* b.hartmann@iu-bremen.de

**Jon Urrestilla**
*Department of Physics and Astronomy, University of Sussex*
*Brighton, U.K.*
*E-mail:* kap10@sussex.ac.uk

Abstract: We study static, spherically symmetric, composite global-local monopoles with a direct interaction term between the two sectors in the regime where the interaction potential is large. At some critical coupling the global defect disappears and with it the deficit angle of the space-time. We find new solutions which represent local monopoles in an Anti-de-Sitter spacetime. In another parameter range the magnetic monopole, or even both, disappear. The decay of the magnetic monopole is accompanied by a dynamical transition from the higgsed phase to the gauge-symmetric phase. We comment on the applications to cosmology, topological inflation and braneworlds

Keywords: Solitons Monopoles and Instantons, Spontaneous Symmetry Breaking, Black Holes.

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## 1. Introduction

The properties and interactions of branes and D-strings have few analogues in field theory and those analogues are by no means perfect. In this respect, the interplay of global and local symmetries can provide very useful toy models [1, 2]. Here we consider one such example in which issues like the decay of deficit angles, gauge symmetry restoration and a dynamical reduction of the effective cosmological constant (as would be needed, for instance, to exit inflation) can be investigated analytically and numerically. The model has $\mathrm{SU}(2)_{\mathrm{global}} \times \mathrm{SU}(2)_{\mathrm{gauge}}$ symmetry with a direct coupling between the two scalar sectors and in a certain range of parameters it admits a composite defect made of a magnetic monopole and a global monopole (a hedgehog).

Defects consisting of a local monopole and a global monopole interacting solely through gravity were considered by Olasagasti [3] in the $\sigma$-model approximation. The full field theoretical solutions coupled to gravity were given in [4, 5] for the case of zero or weak coupling, respectively. Here we investigate the limit of strong coupling.

The stability of such composite systems is unclear [6, 7] but here we take the point of view, inspired by braneworlds, that the cores are fixed at one particular point in space (this would be the case, for instance, if the branes were localized at the minimum of some potential in the higher dimensional space). We investigate the static, spherically symmetric solutions.

Magnetic monopoles were first introduced by 't Hooft and Polyakov [8]. In flat space they have a size roughly of order the inverse vector mass $e\eta_1$, where $e$ is the SU(2) coupling constant and $\eta_1$ is the v.e.v. of the Higgs field. The monopoles carry a quantised magnetic charge $2\pi n/e$, where $n$ is the winding number of the monopole. The 't Hooft-Polyakov monopole in curved space-time was first studied in [9], and a Reissner-Nordström type solution to the coupled system of equations was found. After [10] gave hints that globally regular, gravitating monopole solutions should exist, these solutions were constructed numerically in [11−13]. It was shown that these gravitating monopole solutions exist only for small monopole masses. When the Schwarzschild radius of the solution $\propto \eta_1 G/e$ (where $G$ is Newton's constant) is comparable to the size of the monopole $\propto (\eta_1 e)^{-1}$, the monopoles become black holes which outside of the horizon correspond to the Reissner-Nordström solution and are thus uniquely characterised by their charge and mass. On the other hand, mini-black holes sitting inside the core of the monopoles are also possible [12, 13]. These — in contrast to the Reissner-Nordström solutions — have non-trivial matter fields outside their horizon and thus violate the no-hair conjecture.

In flat space, global monopoles have a logarithmically divergent energy coming from the gradients of the scalar field far from the core, and their stability with respect to angular collapse has been a matter of confusion for some years [14], possibly because global monopoles are stable to infinitesimal axisymmetric perturbations, but the spherical configuration can be deformed with finite *extra* energy to a new decay channel [15]. In [16], it has been demonstrated that the gravitating global monopoles are stable against spherical as well as polar perturbations. When gravitational effects are considered, as shown by Barriola and Vilenkin, global monopoles have a solid deficit angle [17] $\delta \propto G\eta_2^2$ if $\eta_2$ is the vacuum expectation value of the scalar field forming the monopole. In this paper we consider solutions with deficit angle smaller than $4\pi^2$ ($8\pi G\eta_2^2 < 1$). For larger deficit angles there are no static solutions, and in particular there can be *topological inflation* at the monopole core [18, 19].

The composite local-global monopole solutions with spherical symmetry were studied in detail by Spinelly et al [4]. The spherically symmetric composite monopoles show a number of interesting features: in [4] the system with only "indirect" interaction via gravity was studied. It was found that far from the core of the composite defect, the space-time corresponds to a Reissner-Nordström space-time with solid deficit angle.

An interaction term between the Higgs field of the magnetic monopole and the scalar field that forms the global monopole was introduced shortly afterwards, and again the spherically symmetric solutions were analysed in [5, 20] in the case of weak interaction. The purpose of this paper is to show that, when the interaction term is sufficiently large, there is a major qualitative change in the nature of the defects and the surrounding spacetime as the defects become unstable and disappear.

The basic idea in the monopole model analysed below is the following. The model contains gravitational and gauge fields and two sets of scalars. The defect is a composite defect made of a topological global monopole giving a solid deficit angle to the spacetime around it, and a topological gauged ('t Hooft- Polyakov) one giving a long-range magnetic field. The vacuum manifold is $S^2 \times S^2$. An interaction term between the two sets of scalars is introduced. The interaction term couples the scalars and is such that the composite defect is in a spacetime with no cosmological constant (other choices are possible and they will be considered elsewhere).

For low coupling, the main changes are to the detailed profiles and spacetime structures around the defect, and were analysed in [5]. In particular the mass that one assigns to the composite becomes positive in certain regimes (as opposed to negative for the bare global monopole).

For critical values of the scalar couplings, however, a drastic change occurs as the vacuum manifold changes to $S^5$ and the monopoles become non-topological. At this point it becomes energetically favourable for the global monopole to decay, thus removing the linearly divergent energy due to the slow fall-off of scalar gradients far from the core. The solid deficit angle of the solution disappears with the global monopole, and the 't Hooft-Polyakov monopole changes its core structure and mass (but keeps the same magnetic charge).

The interaction coupling can be increased further. In that case we find two possibilities, depending on parameters: either we recover the AdS monopole solutions of [21, 22] — that is, the stable defects are magnetic monopoles in a spacetime with negative cosmological constant — or the system may also be able to decay to the true vacuum and thus get rid of both monopoles.

Our paper is organised as follows: in section 2, we give the model including the spherically symmetric Ansatz, the equations of motion and the boundary conditions. In section 3, we discuss analytic results for the defects arising in the limit of large coupling. In section 4, we present our numerical results and in section 5, we summarize and discuss our results.

## 2. The model

The action of the composite defects' system that we will consider reads [5]:

$$S = S_G + S_M = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G}R + \mathcal{L}_{\text{local}} + \mathcal{L}_{\text{global}} + \mathcal{L}_{\text{inter}}\right), \tag{2.1}$$

where $R$ is the Ricci scalar, $G$ is Newton's constant and $\mathcal{L}_{\text{local}}$, $\mathcal{L}_{\text{global}}$ and $\mathcal{L}_{\text{inter}}$ denote, respectively, the lagrangian density of the local defect, the global defect and the interaction potential, which couples the two sectors directly.

We have:

$$\begin{aligned}\mathcal{L}_{\text{local}} &= -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu,a} - \frac{1}{2}(D_\mu\phi^a)(D^\mu\phi^a) - \frac{\lambda_1}{4}\left(\phi^a\phi^a - \eta_1^2\right)^2 , \\ \mathcal{L}_{\text{global}} &= -\frac{1}{2}(\partial_\mu\chi^a)(\partial^\mu\chi^a) - \frac{\lambda_2}{4}\left(\chi^a\chi^a - \eta_2^2\right)^2\end{aligned} \tag{2.2}$$

and

$$\mathcal{L}_{\text{inter}} = -\frac{\lambda_3}{2}\left(\phi^a\phi^a - \eta_1^2\right)\left(\chi^a\chi^a - \eta_2^2\right), \tag{2.3}$$

with $a = 1, 2, 3$, and $\phi^a$ and $\chi^a$ are scalar triplets. This makes the total potential the most general quartic potential invariant under $\mathrm{SU}(2)_{\text{gauge}} \times \mathrm{SU}(2)_{\text{global}}$ up to an additive constant whose effect is to change the value of the cosmological constant. Here we choose the constant to be zero when $|\phi^a| = \eta_1$, $|\chi^a| = \eta_2$, which means that in the range of parameters where the composite monopole solution exists, the cosmological constant is zero.

The field strength tensor and covariant derivative of the Higgs field read:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\nu - e\epsilon_{abc}A^b_\mu A^c_\nu\,, \qquad D_\mu\phi^a = \partial_\mu\phi^a - e\epsilon_{abc}A^b_\mu\phi^c\,, \tag{2.4}$$

where $A_\mu$ is a SU(2) gauge field and $e$ is the gauge coupling constant.

### 2.1 The structure of the vacuum manifold

Consider the scalar potential

$$V(\phi,\chi) = \frac{\lambda_1}{4}\left(\phi^a\phi^a - \eta_1^2\right)^2 + \frac{\lambda_2}{4}\left(\chi^a\chi^a - \eta_2^2\right)^2 + \frac{\lambda_3}{2}\left(\phi^a\phi^a - \eta_1^2\right)\left(\chi^a\chi^a - \eta_2^2\right). \tag{2.5}$$

Global stability of $V$ requires $\lambda_1 > 0$, $\lambda_2 > 0$. $\lambda_3$ can have either sign, but stability requires $\lambda_3 > -\sqrt{\lambda_1\lambda_2}$.

When $\lambda_3 > 0$ and $\Delta = \lambda_1\lambda_2 - \lambda_3^2 > 0$ the vacuum manifold is the same as in the non-interacting ($\lambda_3 = 0$) case, $S^2 \times S^2$, given by

$$\phi^a\phi^a = \eta_1^2\,, \qquad \chi^a\chi^a = \eta_2^2\,. \tag{2.6}$$

This case was analysed in great detail in [5].

However, if $\Delta = \lambda_1\lambda_2 - \lambda_3^2 = 0$ ($\lambda_3 > 0$) the vacuum manifold becomes topologically equivalent to $S^5$ as can be seen from the identity

$$V = \frac{1}{4}\left[\sqrt{\lambda_1}(\phi^a\phi^a - \eta_1^2) + \sqrt{\lambda_2}(\chi^a\chi^a - \eta^2)\right]^2 + \frac{1}{2}\left[\lambda_3 - \sqrt{\lambda_1\lambda_2}\right]\left(\phi^a\phi^a - \eta_1^2\right)\left(\chi^a\chi^a - \eta_2^2\right). \tag{2.7}$$

The second term is identically zero and the vacuum states are now given by the condition

$$\sqrt{\lambda_1}\phi^a\phi^a + \sqrt{\lambda_2}\chi^a\chi^a = \sqrt{\lambda_1}\eta_1^2 + \sqrt{\lambda_2}\eta_2^2\,. \tag{2.8}$$

The monopoles are no longer topological. We call this the *critical coupling* case.

We can consider even larger values of $\lambda_3$. In that case, $\lambda_3 > \sqrt{\lambda_1\lambda_2}$, there are two possible minima of the potential

$$\phi^a = 0\,, \qquad \chi^a\chi^a = \eta_2^2 + \eta_1^2\frac{\lambda_3}{\lambda_2} \qquad \text{with} \qquad V_f = \frac{\eta_1^4}{4\lambda_2}\Delta \tag{2.9}$$

and

$$\phi^a\phi^a = \eta_1^2 + \eta_2^2\frac{\lambda_3}{\lambda_1}\,, \qquad \chi^2 = 0 \qquad \text{with} \qquad V_h = \frac{\eta_2^2}{4\lambda_1}\Delta\,. \tag{2.10}$$

In this regime the composite monopole is unstable. The potential has two minima with different (negative) values, and we might expect the lowest one to be favoured if there is no winding. In general, (2.9), respectively (2.10) is the lowest minimum for $\eta_1^4\lambda_1 > (<)\ \eta_2^4\lambda_2$. However, the potential energy "criterion" is not enough to predict what will be the static solution. Actually, numerical simulations show that the static solution has $\chi^a = 0$ for all cases, due to the linearly divergent gradient energy of the global monopole (even at $\Delta = 0$). So the global monopole disappears for all choices of parameters. The solution then is a static solution corresponding to a magnetic monopole in Anti-de-Sitter space (see below for more details). The monopole core size and asymptotic field values change according to the new vacuum expectation value of the Higgs field.



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For the case $\lambda_3 < 0$ and $\Delta = \lambda_1\lambda_2 - \lambda_3^2 > 0$ the situation is analogous to the previous case. The solutions tend asymptotically to (2.6), with some changes in the profile, as we will show in section 4.

On the other hand for $\lambda_3 < 0$ and $\Delta = 0$, the potential becomes

$$V = \frac{1}{4}\left[\sqrt{\lambda_1}(\phi^a\phi^a - \eta_1^2) - \sqrt{\lambda_2}(\chi^a\chi^a - \eta_2^2)\right]^2 \tag{2.11}$$

with minima

$$\sqrt{\lambda_1}\phi^a\phi^a - \sqrt{\lambda_2}\chi^a\chi^a = \sqrt{\lambda_1}\eta_1^2 - \sqrt{\lambda_2}\eta_2^2\,. \tag{2.12}$$

The existence of different solutions depends strongly on the values of the parameters. Contrary to the previous case, it might be possible that the global monopole "survives", while the local monopole disappears. Let us be more specific:

If $\sqrt{\lambda_1}\eta_1^2 < \sqrt{\lambda_2}\eta_2^2$, then the global monopole will not disappear, because equation (2.12) cannot be fulfilled for $\chi = 0$; but the local one can disappear. Likewise, the possibility of having the global monopole disappear and the local remain persists for $\sqrt{\lambda_1}\eta_1^2 > \sqrt{\lambda_2}\eta_2^2$.

This is somewhat analogous to what happens in D-term N=1 supersymmetric models with a Fayet-Iliopoulos (FI) term. Introducing $\kappa_{\mathrm{FI}} \equiv \sqrt{\lambda_1}\eta_1^2 - \sqrt{\lambda_2}\eta_2^2$, equation (2.11) can be rewritten suggestively as

$$V = \frac{1}{4}\left[\sqrt{\lambda_1}|\phi|^2 - \sqrt{\lambda_2}|\chi|^2 - \kappa_{\mathrm{FI}}\right]^2\,. \tag{2.13}$$

Static defects (vortices) in models with such a potential were studied in [23, 24]. One of the fields is identically zero everywhere, and the field that vanishes is given by the sign of $\kappa_{\mathrm{FI}}$: if $\kappa_{\mathrm{FI}} > 0$ then $\chi = 0$; and if $\kappa_{\mathrm{FI}} < 0$ then $\phi = 0$.

When the magnetic monopole decays, the magnetic field becomes weaker at the core and the core expands to an infinite size. At any stage of the decay, the total magnetic flux integrated on a sphere sufficiently far from the monopole core is constant, but there is no localized object carrying this magnetic flux. The end result is indistinguishable from Minkowski space except for the conserved magnetic flux that has been diluted to an unobservable level. (The situation is completely analogous to the semilocal string model in the unstable regime, where there is a conserved flux which nevertheless spreads over an infinite area and is not carried by any localized vortex [25 – 27]).

There is a boundary case $\sqrt{\lambda_1}\eta_1^2 = \sqrt{\lambda_2}\eta_2^2$, where *both* monopoles disappear at the same time, and the magnetic charge gets diluted (the analogous case would be supersymmetric QED without FI term, where the possible vortices were shown to be unstable and decay to the vacuum [28]).

### 2.2 Spherically symmetric Ansatz

In what follows we consider configurations with spherical symmetry, with the two monopole cores fixed at the origin. It is not obvious that this assumption is justified dynamically, since the stability of the composite monopole is still unclear [6, 7] and it is possible that the cores of the two monopoles will expel each other, separating the global and local monopoles. Nevertheless we are interested in a possible application to braneworlds, where the higher dimensional theory might provide a pinning potential (not manifest in the low energy theory) which keeps the cores at the origin.[1]

The Ansatz for the spherically symmetric metric tensor in Schwarzschild-like coordinates reads:

$$ds^2 = -A^2(r)N(r)dt^2 + N^{-1}(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\,, \tag{2.14}$$

where we define for later convenience the mass function $m(r)$ as follows :

$$N(r) = 1 - 8\pi G\eta_2^2 - \frac{2m(r)}{r}\,. \tag{2.15}$$

Note that in the case of spherical symmetry the only choice for the winding number is $n = 1$ or $n = 0$. Higher winding number configurations would have axial or even less symmetry. We consider a composite monopole with unit winding in both the global and local sectors. The Ansatz for the static spherically symmetric global field $\chi^a$, the Higgs field $\phi^a$ and the gauge field $A^a_\mu$ in cartesian coordinates reads:

$$\phi^a(x) = \eta_1 h(r)\hat{x}^a\,, \qquad \chi^a(x) = \eta_1 f(r)\hat{x}^a\,, \tag{2.16}$$

$$A^a_i(x) = \epsilon_{iaj}\hat{x}^j\frac{1-u(r)}{er}\,, \qquad A^a_0(x) = 0\,. \tag{2.17}$$

For convenience we have rescaled the global field with the vacuum expectation value of the local field. Note that the configuration given by a $n = 1$ magnetic monopole and

$$\chi^a = \eta_1 f(r){n_\chi}^a \qquad {n_\chi}^a{n_\chi}^a = 1\,, \tag{2.18}$$

with $n^a_\chi$ a fixed direction in internal space, is also spherically symmetric. Similarly, the $n = 1$ global monopole

$$\phi^a(x) = \eta_1 h(r){n_\phi}^a\,, \qquad A^a_\mu(x) = 0\,, \tag{2.19}$$

with $n^a_\chi$ fixed is also spherically symmetric. The same is true of the ansatz given by the previous two equations (2.18), (2.19), which has zero winding in both sectors.

### 2.3 Equations of motion

Varying (2.1) with respect to the matter fields and gravitational fields and introducing the dimensionless variable $x$ and dimensionless mass function $\mu(x)$ [5]:

$$x = e\eta_1 r\,, \qquad \mu(x) = e\eta_1 m(r) \tag{2.20}$$

as well as the dimensionless coupling constants:

$$q = \frac{\eta_2}{\eta_1}\,, \qquad \gamma^2 = 8\pi G\eta_1^2\,, \qquad \beta_i = \frac{\lambda_i}{e^2}\,, \qquad i = 1, 2, 3 \tag{2.21}$$

[1] For $\lambda_3 > 0$ the cores repel, but even for $\lambda_3 \leq 0$ there may be a gravitational instability [6, 7].

we obtain the following set of differential equations (the prime denotes the derivative with respect to $x$):

$$\begin{aligned}
\left[x^2ANh'\right]' &= A\left[2u^2h + x^2\beta_1(h^2-1)h + x^2\beta_3 h(f^2-q^2)\right], && (2.22)\\
\left[x^2ANf'\right]' &= A\left[2f + x^2\beta_2 f(f^2-q^2) + x^2\beta_3(h^2-1)f\right], && (2.23)\\
\left[ANu'\right]' &= A\left[\frac{u(u^2-1)}{x^2} + uh^2\right], && (2.24)\\
(xAN)' &= [1-\gamma^2x^2\bar{U}]A, && (2.25)\\
A' &= \gamma^2 Ax\bar{K}, && (2.26)
\end{aligned}$$

where we have defined

$$\begin{aligned}
\bar{U} = &\frac{(u^2-1)^2}{2x^4} + \frac{u^2h^2}{x^2} + \frac{f^2}{x^2} + \frac{\beta_1}{4}(h^2-1)^2 + \frac{\beta_2}{4}(f^2-q^2)^2 + \\
&+ \frac{\beta_3}{2}(h^2-1)(f^2-q^2), \qquad (2.27)
\end{aligned}$$

and

$$\bar{K} = \frac{1}{2}\left(\frac{df}{dx}\right)^2 + \frac{1}{2}\left(\frac{dh}{dx}\right)^2 + \frac{1}{x^2}\left(\frac{du}{dx}\right)^2. \qquad (2.28)$$

Moreover, using the definition (2.15), eq. (2.25) can be further brought to the form

$$\mu' = \frac{1}{2}\gamma^2x^2\left(N\bar{K} + \left(\bar{U} - \frac{q^2}{x^2}\right)\right). \qquad (2.29)$$

## 3. Exotic defects and changed asymptotic values

As explained earlier (see also [5]), the total rescaled potential

$$\begin{aligned}
\hat{V} &= \left[\frac{\lambda_1}{4}\left(\phi^a\phi^a-\eta_1^2\right)^2 + \frac{\lambda_2}{4}\left(\chi^a\chi^a-\eta_2^2\right)^2 + \frac{\lambda_3}{2}\left(\phi^a\phi^a-\eta_1^2\right)\left(\chi^a\chi^a-\eta_2^2\right)\right]\frac{1}{e^2\eta_1^4}\\
&= \frac{\beta_1}{4}(h^2-1)^2 + \frac{\beta_2}{4}(f^2-q^2)^2 + \frac{\beta_3}{2}(h^2-1)(f^2-q^2) \qquad (3.1)
\end{aligned}$$

has different properties according to the sign of $\Delta = \lambda_1\lambda_2 - \lambda_3^2$ (which equals the sign of $\hat{\Delta} := \beta_1\beta_2 - \beta_3^2$).

For $\Delta > 0$ the potential is semi-positive definite and its minima are attained for $h^2 = 1$ and $f^2 = q^2$. In this case, requiring that the Higgs and Goldstone field go to their respective vacuum values ensures that the asymptotic value of the potential energy density is zero.

For $\Delta < 0$ and $\beta_3 > 0$, the configuration $h^2 = 1$ and $f^2 = q^2$ becomes a saddle point and the two new minima occur for

$$h^2 = 0, \qquad f^2 = \left(q^2 + \frac{\beta_3}{\beta_2}\right) = f^2_{\min} \qquad (3.2)$$

and

$$f^2 = 0, \qquad h^2 = \left(1 + \frac{\beta_3}{\beta_1}q^2\right) = h^2_{\min}. \qquad (3.3)$$

For the case $\Delta = 0$ there is a whole family of minima for

$$h^2 = 1 - \frac{\beta_3}{\beta_1}\left(f^2 - q^2\right) \quad \text{or, equivalently,} \quad \sqrt{\beta_1}(h^2 - 1) = \pm\sqrt{\beta_2}(q^2 - f^2)\,, \tag{3.4}$$

where the $\pm$ corresponds to the sign of $\beta_3$.

The boundary conditions used previously in the literature (in [5] for instance)

$$u(x = 0) = 1\,, \qquad f(x = 0) = h(x = 0) = \mu(x = 0) = 0 \tag{3.5}$$

and

$$u(x = \infty) = 0\,, \qquad A(x = \infty) = 1 \tag{3.6}$$

$$h(x = \infty) = 1\,, \qquad f(x = \infty) = q \tag{3.7}$$

are clearly not suitable for the range of possibilities we encounter in the present case. Therefore, we adopt a different strategy: instead of fixing the values of the fields $f$ and $h$ at infinite, we impose that the *derivative with respect to* $x$ of the fields $f(x)$ and $h(x)$ should vanish at infinity. The system is left "free to choose" the asymptotic values of the fields. We then confirm that the values chosen are the correct ones.

Our boundary conditions are thus equal to the ones given above, except for the values of $h$ and $f$ at infinity: the boundary conditions are given by conditions (3.5), (3.6), but conditions (3.7) are replaced by

$$h'(x)|_{x=\infty} = 0\,, \qquad f'(x)|_{x=\infty} = 0\,. \tag{3.8}$$

As we confirmed numerically, the asymptotic behaviours for the fields $h$ and $f$ for $\Delta \leq 0$ are the ones given in (3.3) (see below for more details). We could also impose $u'(x)|_{x=\infty} = 0$ instead of $u(x = \infty) = 0$.

### 3.1 From deficit angle to cosmological constant

The different asymptotic values of the scalar fields have drastic consequences on the metric of the system. For $\Delta > 0$, the vacuum values of $f$ and $h$ ($f^2 = q^2$, $h^2 = 1$) correspond to zero potential, whereas for $\Delta < 0$ the minimum value of the potential is negative and results in a cosmological constant. The deficit angle of the global monopole is also tied to the asymptotic values of the scalars. Therefore, we find different types of space-times in this model depending on the value of $\Delta$ and the sign of $\beta_3$.

If $\Delta > 0$ (irrespective of the sign of $\beta_3$), from (2.29) we have

$$\mu'|_{x\gg 1} \sim \frac{1}{2}\gamma^2 x^2 \left(\frac{1}{2x^4}\right) \tag{3.9}$$

leading to

$$N(x \gg 1) \sim 1 - \gamma^2 q^2 - 2\frac{\mu_\infty}{x} + \frac{\gamma^2}{2x^2} + O\left(x^{-3}\right) \tag{3.10}$$

with $\mu_\infty$ being the integration constant. This represents a space-time with deficit solid angle equal to $\gamma^2 q^2$ (times $4\pi^2$) and magnetic charge equal to unity (in units of $2\pi/e$).

However, for $\Delta < 0$ and $\beta_3 > 0$ the fields behave asymptotically like (3.2) or (3.3). We found numerically that the preferred solution is given by (3.3), since that corresponds to setting the global monopole (which has divergent energy) to zero.

Thus, we find the following behaviour for the metric

$$\mu'|_{x\gg 1} \sim \frac{\gamma^2}{2} x^2 \frac{q^4}{4}\left(\beta_2 - \frac{\beta_3^2}{\beta_1^2}\right) - \frac{\gamma^2}{2} q^2 + \frac{\gamma^2}{4x^2} \tag{3.11}$$

which gives for $N(x)$ :

$$N(x \gg 1) \sim 1 - 2\frac{\mu_\infty}{x} + \frac{\gamma^2}{2x^2} + \frac{\gamma^2}{2} q^4 \left(\frac{\beta_3^2}{\beta_1} - \beta_2^2\right)\frac{x^2}{6} = 1 - 2\frac{\mu_\infty}{x} + \frac{\gamma^2}{2x^2} - \frac{\gamma^2 q^4 \Delta}{\beta_1^2}\frac{x^2}{12}\,. \tag{3.12}$$

Note that the coefficient of the $x^2$ term is identified with (minus one third of) the cosmological constant. This metric thus represents an Anti-de-Sitter space-time without deficit angle and with negative cosmological constant $\Lambda$:

$$\Lambda := \frac{\gamma^2 q^4 \Delta}{4\beta_1^2}\,. \tag{3.13}$$

For $\Delta = 0$ and $\beta_3 > 0$, the asymptotic value of $f$ and $h$ follow equation (3.4), so we find that

$$\begin{aligned} \mu'|_{x\gg 1} &\sim \frac{1}{2}\gamma^2(f^2 - q^2) + \frac{\gamma^2}{4x^2} \qquad \longrightarrow \\ N(x \gg 1) &\sim 1 - \gamma^2 q^2 + \frac{\gamma^2}{2x^2} - \gamma^2(f^2 - q^2) - 2\frac{\mu_\infty}{x}\,. \end{aligned} \tag{3.14}$$

Out of all the possible solutions (3.4), the system chooses the one with $f = 0$, since that minimises the energy, so we have (using the definition of $\gamma$ and $q$) that the solid deficit angle cancels out, and the metric just represents an asymptotically flat metric without deficit angle, a Reissner-Nordström space-time.

For $\Delta = 0$ and $\beta_3 < 0$ the solutions depend on the parameters. For $\sqrt{\beta_1} > \sqrt{\beta_2} q^2$ the situation is analogous to the case with $\beta_3 > 0$ (3.14).

However, for $\sqrt{\beta_1} < \sqrt{\beta_2} q^2$ the solution is $h = 0$ and $f^2 = q^2 - \sqrt{\frac{\beta_1}{\beta_2}}$, so the metric functions are now:

$$\begin{aligned} \mu'|_{x\gg 1} &\sim \frac{\gamma^2}{4x^2} - \frac{1}{2}\gamma^2\sqrt{\frac{\beta_1}{\beta_2}} \longrightarrow \\ N(x \gg 1) &\sim 1 - \gamma^2\left(q^2 - \sqrt{\frac{\beta_1}{\beta_2}}\right) + \frac{\gamma^2}{2x^2} - 2\frac{\mu_\infty}{x}\,. \end{aligned} \tag{3.15}$$

This corresponds to a space-time with deficit angle $\propto (q^2 - \sqrt{\frac{\beta_1}{\beta_2}})$ and magnetic charge equal to unity.

These cases will be illustrated by numerical simulations in section 4.

### 3.2 From higgsed phase to symmetric phase

The AdS solution obtained in the previous section for $\Delta < 0$ and $\beta_3 > 0$ was a consequence of the new boundary conditions given by equations (3.5), (3.6) and (3.8). But one could also wonder if that is the best choice of boundary conditions.

We have to bear in mind that we are actually analysing *static* solutions for the problem given some values of the parameters. But we could try to imagine a situation in which we begin with some fixed values of the parameters, and $\beta_3$ varies from $\beta_3 = 0$ to higher values; which would correspond to $\Delta$ ranging from $\Delta > 0$ to $\Delta < 0$ passing through $\Delta = 0$.

However, we have seen that at $\Delta = 0$, the global monopole disappears. Therefore, the Ansatz for the global scalar function together with its boundary conditions might not be optimal for this case. Let us consider the case where the conditions for $\chi$ are changed to an Ansatz with no winding

$$\chi^a = \eta_1 f(r)\,. \tag{3.16}$$

The equation of motion for $f$ (2.23) will change to

$$\left[x^2 A N f'\right]' = A\left[x^2\beta_2 f(f^2 - q^2) + x^2\beta_3(h^2 - 1)f\right]. \tag{3.17}$$

There is no need of imposing $f(0) = 0$ now, so we can change that to $f'(0) = 0$ also. We then obtain a different solution for the system when $\Delta < 0$. It can be seen analytically that the configuration

$$f(x) = f_{\min}\,, \qquad h(x) = 0\,, \qquad u(x) = 1\,, \qquad \mu(x) = 0 \tag{3.18}$$

is a solution. This corresponds to having all the windings disappear, the Higgs mechanism ceases to operate, and we are left in the true vacuum. The value of the constant $f_{\min}$ is the one that minimizes the potential (3.2).

Once the Higgs field has vanished everywhere, the Ansatz and boundary conditions for $\phi$ should also be revised, and changed as in the $\chi$ case. So the system would in principle choose the solution corresponding to the lowest minimum (2.9) or (2.10).

Of course, the question arises of how these processes would take place in a dynamical setting, and whether the system would evolve toward one of these solutions getting rid of all the topology altogether. In physical terms one can imagine the formation of a condensate of the global scalar field whose effect is to lower the value of $f$ and expand the magnetic core. Once $f$ is zero in a sufficiently large region a condensate of $\phi$ with no winding can develop at the expense of the global condensate and eventually reach the solution with $f = f_{\min}$, $h = 0$. We will investigate this process in a subsequent publication.

## 4. Numerical results

We have solved the equations (2.23)–(2.26) subject to the boundary conditions (3.5), (3.6) and (3.8) for different values of the parameters using relaxation methods and the damped Newton method of quasi-linearization [29].

We first reproduced known results [4, 5] for the case $\Delta > 0$. Despite using different boundary conditions, the system finds the same solution as in previous work [4, 5]. Figure 1

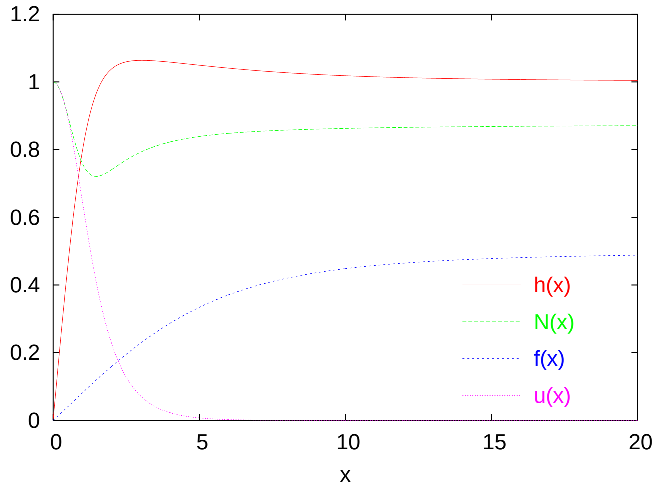


**Figure 1:** The profile of the fields $f(x)$, $h(x)$, $u(x)$ and $N(x)$ for the composite monopole with $\beta_1 = \beta_2 = 1$, $\beta_3 = 0.75$ (thus, $\Delta \sim 0.44$), $q = 0.5$ and $\gamma^2 = 0.5$.

shows the typical type of profile obtained for the case $\Delta > 0$: the Higgs field $h$ grows higher than its asymptotic value and then goes down to 1. The global field $f$ tends to its vacuum value $q$ more slowly than in a non-coupled case.

In figure 2 we show the profiles of the function $h$ for different values of $\beta_3$, always in the regime $\Delta > 0$. In all those cases, the profile of $f$ does not vary much (not shown). Comparing to the non-interacting case $\beta_3 = 0$, the function $h$ goes to higher (lower) values around the core for $\beta_3$ positive (negative). The reason could be that the system "sees" the new minimum ($f = 0, h = h_{\rm min}$), and in places where $f \ll q$, it tries to go to that minimum. Moreover, around the core of the defect the metric function is usually small, and thus, gradient energy becomes "cheaper" than potential energy.

On the other hand, when $\Delta \leq 0$ ($\beta_3 > 0$) the solutions are very different: the function $f(x) \equiv 0$ and the function $h(x)$ tends asymptotically to the value given in (3.3), i.e. the global monopole disappears and the Higgs field asymptotes to its new vacuum expectation value. This happens irrespective of the values of the parameters $q$, $\beta_1$ and $\beta_2$, even if $V_h < V_f$. This is the case also for $\Delta = 0$, where all the possible minima are degenerate in *potential* energy; but it is obviously favourable to set the global monopole to zero, since it has divergent gradient energy.

In order to show that the global monopole is the one that disappears, we include figures 3 and 4. In figure 3 a) and b) we plot the functions $f$ and $h$ for different values of the parameters. One set of curves in each figure corresponds to $\Delta > 0$ (to have the composite defect as a reference) and the other set to $\Delta = 0$. The case with $\Delta < 0$ is analogous to $\Delta = 0$. Figure 3*a* has $\sqrt{\frac{\beta_1}{\beta_2}} < q^2$ and b) has $\sqrt{\frac{\beta_1}{\beta_2}} > q^2$. In figure 4 we present a case with $q > 1$ ($\eta_1 = 0.5$ and $\eta_2 = 1$) to show that the global monopole also disappears in this case.

Furthermore, the metric function $N(x)$ asymptotically tends to the Anti-de-Sitter form given in (3.12). This is demonstrated in figure 5 and figure 6 for $\beta_1^2 = \beta_2^2 = 1$, $\gamma^2 = 0.5$,

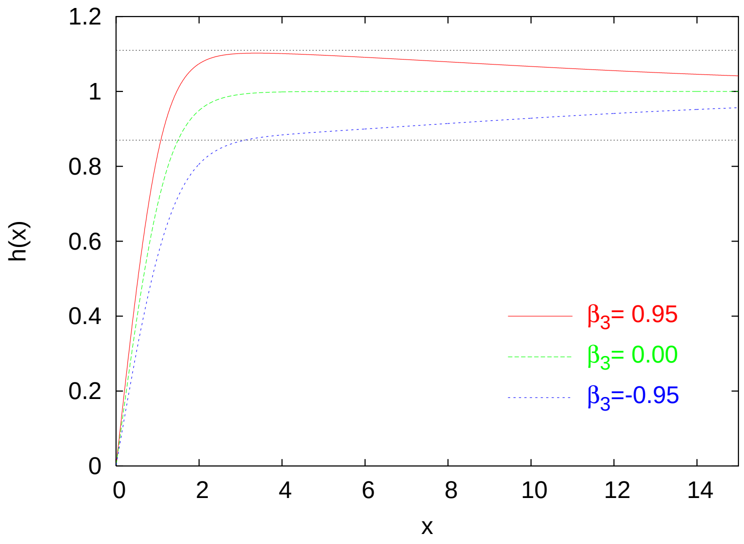


**Figure 2:** Profiles of the function $h$ for $\beta_1 = \beta_2 = 1$, $q = 0.5$, $\gamma^2 = 0.5$ and different values of $\beta_3$. The horizontal lines shown correspond to $h_{\rm min}(\beta_3 = 0.95) \sim 1.11$ and $h_{\rm min}(\beta_3 = -0.95) \sim 0.87$ given in equation (3.3). Note that close to the core, the function tends to that minimum.

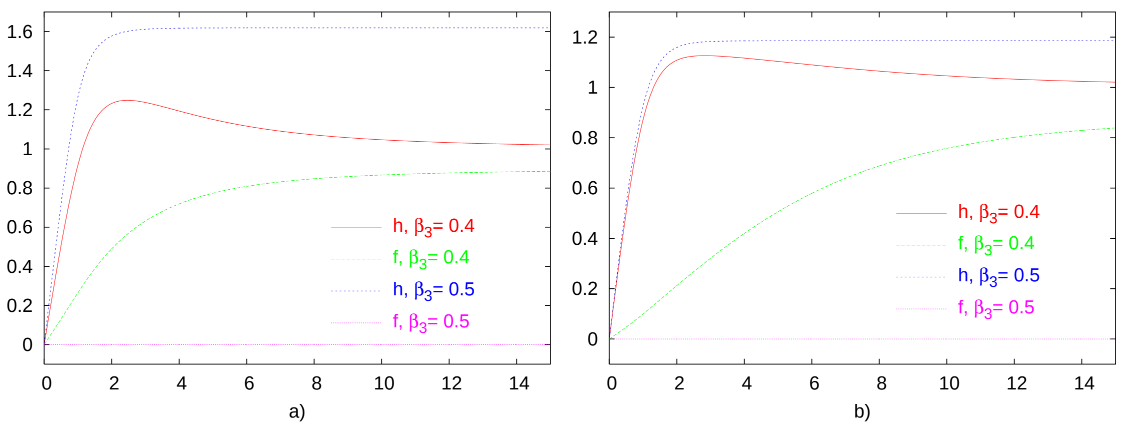


**Figure 3:** $(a)$ Profile of the functions $f$ and $h$ for different values of $\beta_3$, with $\beta_1 = 0.25, \beta_2 = 1$, $q = 0.9$ and $\gamma^2 = 0.5$. In this case $\frac{\beta_1}{\beta_2} < q^4$. $(b)$ Same as before but with $\beta_1 = 1$ and $\beta_2 = 0.25$, so that $\frac{\beta_1}{\beta_2} > q^4$. Note that in both cases, the global field $f$ is the one that vanishes when $\Delta = 0$.

$q = 0.5$ and different values of $\beta_3$. Clearly for $\Delta < 0$ ($\beta_3 > 1$ in this case), $f(x) \equiv 0$ and $h(x)$ tends asymptotically to $h_{\rm min}$ which here is $h_{\rm min} \approx 1.12$ for $\beta_3 = 1$ and $h_{\rm min} \approx 1.15$ for $\beta_3 = 1.25$. Moreover, as can be seen from figure 6, the metric function tends to a value smaller than one for $\Delta > 0$, thus representing a space-time with deficit angle. For $\Delta = 0$, the space-time becomes asymptotically flat, while for $\Delta < 0$, the metric function $N(x)$ rises following a power law representing asymptotically an Anti-de-Sitter–Reissner-Nordström (AdSRN) space-time.

When increasing $\gamma$, the minimum of the metric function gets deeper and finally at some $\gamma = \gamma_{cr}(q, \beta_i)$ a degenerate horizon forms at $x = x_h$ with $N(x_h) = 0$, $N'(x)|_{x=x_h} = 0$.

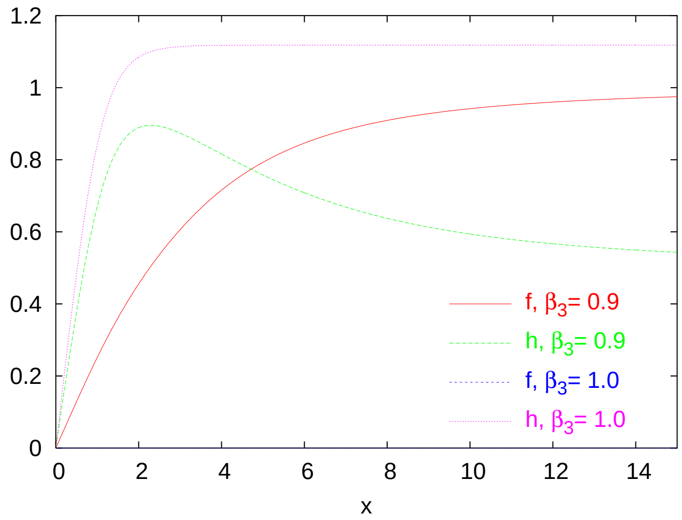


**Figure 4:** Profile of functions $f$ and $h$ for $q > 1$, showing that the global monopole vanishes ($\beta_1 = 1$, $\beta_2 = 1$, $\gamma^2 = 0.5$, $\eta_1 = 0.5$, $\eta_2 = 1$).

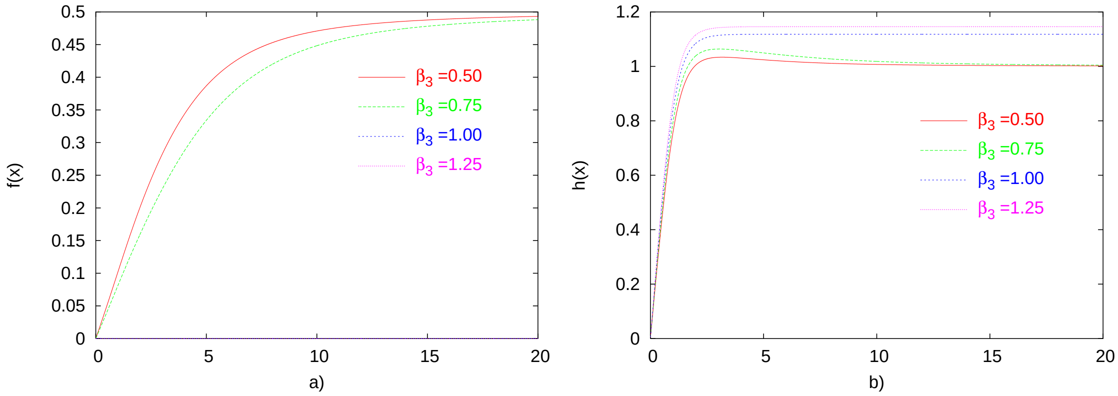


**Figure 5:** The profile of the global field function $f(x)$ $(a)$ and the Higgs field $h(x)$ $(b)$ of the composite monopole system is shown for $\beta_1 = \beta_2 = 1$, $q = 0.5$, $\gamma^2 = 0.5$ and different values of $\beta_3$. Note that $\Delta > 0$ for $\beta_3 = 0.5$ and $\beta_3 = 0.75$, $\Delta = 0$ for $\beta_3 = 1$ and $\Delta < 0$ for $\beta_3 = 1.25$. The profile $f(x) \equiv 0$ for $\Delta \leq 0$. The profile of $h(x)$ also changes with $\Delta$: for $\Delta > 0$ it raises beyond its asymptotic value and goes down; for $\Delta \leq 0$, it goes to its asymptotic value monotonically.

This is demonstrated in figure 7 for $\beta_1 = \beta_2 = 1$, $\beta_3 = 1.25$, $q = 0.5$ and increasing $\gamma$. At the same time the matter functions become equal to their vacuum values outside of this horizon, i.e. $h(x > x_h) \equiv h_{\min}$ and $A(x > x_h) \equiv 1$ with an infinite derivative at $x = x_h$.

The solution thus represents an extremal Anti-de-Sitter–Reissner-Nordström (AdSRN) black hole for $x > x_h$ with

$$N(x > x_h) = 1 - 2\frac{\mu_\infty}{x} - \Lambda\frac{x^2}{3} + \frac{\gamma^2}{2x^2} \tag{4.1}$$

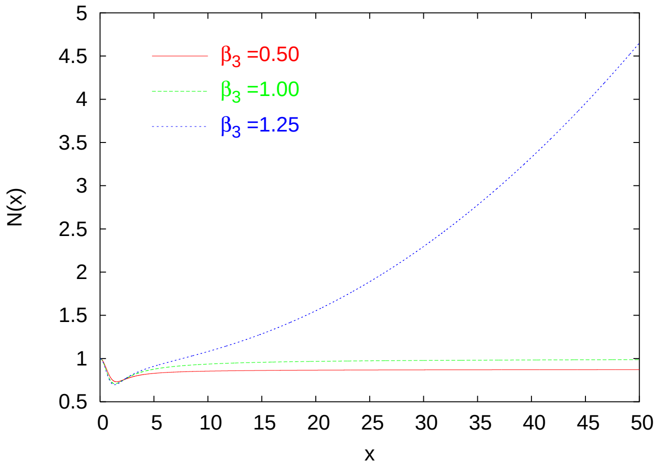


**Figure 6:** Same as figure 5 for the metric function $N(x)$ of the composite monopole system. The powerlaw increase for $\beta_3 > 1$ signals a cosmological constant

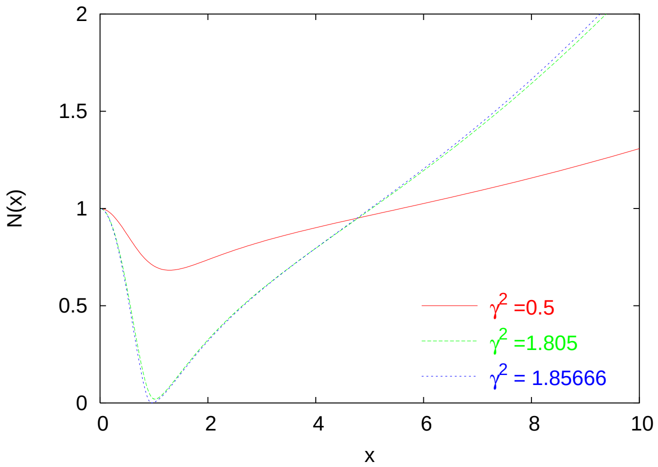


**Figure 7:** The metric function $N(x)$ of the composite monopole system is shown for $\beta_1 = \beta_2 = 1$, $\beta_3 = 1.25$ (thus $\Delta < 0$), $q = 0.5$ and increasing values of $\gamma$.

and extremal horizon at

$$x_h = \frac{1}{\sqrt{2|\Lambda|}}\sqrt{-1+\sqrt{1-2\gamma^2\Lambda}}\,, \tag{4.2}$$

while it is non-trivial and non-singular for $0 \leq x < x_h$. For $\beta_1 = \beta_2 = 1$, $\beta_3 = 1.25$, $q = 0.5$, $\gamma^2 = \gamma_{cr}^2 \approx 1.85666$, we find from the above formula that $x_h \approx 0.94$ which agrees very well with our numerical results.

The case with $\Delta = 0$ and $\beta_3 < 0$ is special. As mentioned above, one or the other (or both) monopoles will disappear depending on the values of the parameters. Thus, for the case $\frac{\beta_1}{\beta_2} > q^4$ the global monopole will disappear (see figure (8) $b$), as in the case with $\beta_3 > 0$. But for $\frac{\beta_1}{\beta_2} < q^4$, the local one disappears (see figure (8) $a$), even if he global one has divergent energy. This is due to the fact that the configuration with $f = 0$ ceases

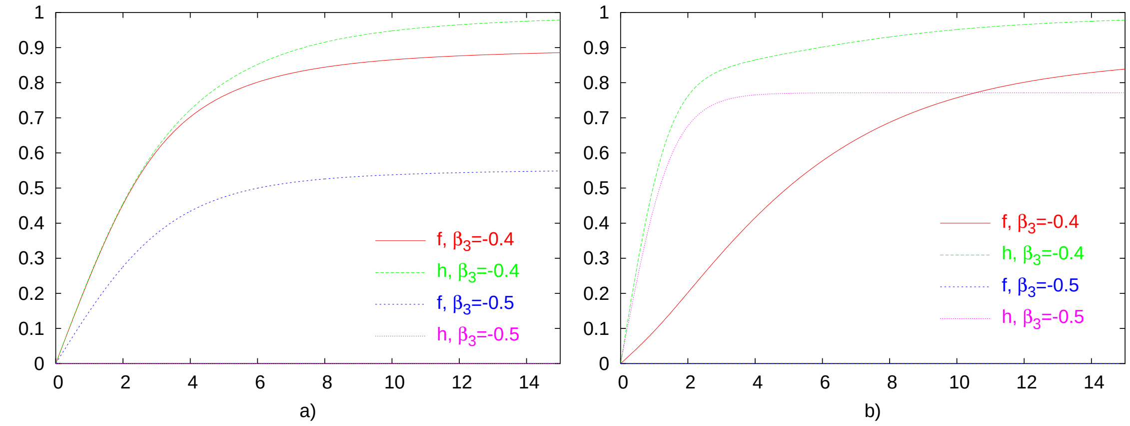


**Figure 8:** This is the equivalent of figure 3 but with $\beta_3 < 0$. Contrary to that case, depending on the value of the parameters the local monopole $f$ (Figure $a$, $\frac{\beta_1}{\beta_2} < q^4$) or the global one $h$ (Figure $b$, $\frac{\beta_1}{\beta_2} > q^4$) will disappear.

to be a solution, so the system does not have the choice of making the global monopole disappear. There are no solutions for $\beta_3 < 0$ and $\Delta < 0$ since the potential is not bounded from below.

## 5. Summary and discussion

Defects containing global and local symmetries have recently become an interesting arena in which one can try to model some of the more exotic properties of superstrings and branes within a field theory context. This is particularly important for applications to cosmology.

In this paper we revisited a simple case, a composite global/local monopole with a quartic interaction term between the two scalar triplets. The static solutions in this model were analysed in [4, 5] for weak coupling, here we have investigated the strong coupling case and discovered a dramatic change in the nature of the solutions.

There is a critical value of the coupling for which both the magnetic monopole and the global monopole become non-topological and potentially unstable. If the interaction coupling constant is positive (corresponding to a repulsive interaction between the cores) we find that the global monopole always disappears. More precisely, for values higher than the critical coupling, we find that the static solution is a local monopole in Anti-de-Sitter space-time which in the limit of sufficiently large gravitational coupling forms a degenerate horizon. Outside of this horizon, the solution then corresponds to an Anti-de-Sitter–Reissner-Nordström solution, while inside it is non-trivial and non-singular. If the interaction term has the opposite sign (core attraction) we find that either or both monopoles can disappear depending on parameters.

We have discussed what are the appropriate boundary conditions in each regime, and how to implement them in numerical calculations. We should stress that all our solutions are in principle constrained, since the condition of spherical symmetry forces both monopole cores to sit at the origin, an assumption that is not always justified dynamically (but that makes sense in the context of applications to braneworlds). It leaves open the question of how a network of such defects would form and evolve in a cosmological context, in particular for what ranges of parameters the two kinds of monopoles would evolve more or less independently and for what ranges they would be tightly coupled (which would enhance magnetic monopole annihilation).

We have concentrated on the nature of the static solutions for various regimes of the couplings, in particular those regimes in which the monopoles become unstable. A dynamical study of how these defects decay is of course very interesting in its own right but there are at least three more reasons why the extension of these results to the time-dependent case should be considered.

First of all, time-dependent solutions with deficit angles larger than $4\pi^2$ can give rise to topological inflation [18, 19] which in this composite model would end naturally when the defects decay (see [30] for a related proposal).

Secondly, an important change in paradigm that has resulted from superstring cosmology and braneworlds is that coupling constants are not truly constant, they are the expectation values of fields and may be space-time dependent. In our particular example the time evolution of one coupling constant (the interaction coupling) could completely change the structure of spacetime. For low coupling the system looks like a magnetic monopole and a global monopole superimposed, and spacetime is asymptotically Reissner-Nordstrom with a deficit angle and no cosmological constant. As the coupling reaches the critical value, the global monopole disappears and with it the deficit angle, and we are left with a magnetic monopole. The magnetic core is unstable and develops a scalar condensate of the global field which makes the core expand indefinitely. In this last step, the $\mathrm{SU}(2)_{\mathrm{gauge}}$ symmetry is restored in the expanding core and the gauge field eventually becomes massless everywhere. The end result is a transition from the Higgs phase to the symmetric phase.

We expect many of the qualitative features found here (decay of deficit angles, cosmological constant reduction, symmetry restoration) to carry over to the composite $\mathrm{U}(1)_{\mathrm{global}} \times \mathrm{U}(1)_{\mathrm{gauge}}$ vortex case, where they should have an application in braneworlds and in the (toy) modelling of orbifold and conifold transitions.

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